Philippine Science High School – Main Campus
Solving Systems of Equations - Substitution
Mathematics 3

- Systems of linear equations can be solved by graphing the lines, and the intersection of the two lines is the solution to the system. Parallel lines have no solutions and equivalent equations have infinitely many solutions.
- Not all equations are easy to graph, and not all solutions have integer coordinates. There is an analytical approach to solving systems of equations (as opposed to the graphical approach.)

To solve for the solution of a system of equations analytically, we use the **Substitution Method**.

1. Using one of the two equations in the system, solve for *y* in terms of *x*. (or solve for x in terms of y)
2. Substitute the equivalent expression of *y* in terms of *x* to the y-variable in the second equation.
3. Solve for the value of *x*. Once you have the value of *x*, solve for the value of *y*.
4. Check your solution by substituting the variables back in the original equations.

Examples: Solve the system of linear equations.

1. $3x + 2y - 4 = 0$ (Equation 1)
   $5x - 2y - 8 = 0$ (Equation 2)

   Step 1: Using one of the two equations in the system, solve for *x* in terms of *y*. (or solve for y in terms of x)

   From Equation 1: $y = -\frac{3}{2}x + 2$

   Step 2: Substitute the equivalent expression of *y* in terms of *x* to the x-variable in the second equation.

   Equation 2: $5x - 2\left(-\frac{3}{2}x + 2\right) - 8 = 0$

   Step 3: Solve for the value of *x*. Once you have the value of *x*, solve for the value of *y*.

   - $5x - 2\left(-\frac{3}{2}x + 2\right) - 8 = 0$
   - $5x + 3x - 4 - 8 = 0$
   - $8x - 12 = 0$
   - $x = \frac{3}{2}$

   From Step 1:

   - $y = \left(-\frac{3}{2}\right)\left(\frac{3}{2}\right) + 2$
   - $y = -\frac{9}{4} + 2$
   - $y = -\frac{1}{4}$

   The solution is $\boxed{\left(\frac{3}{2}, -\frac{1}{4}\right)}$ Check this solution in the original system of linear equations.

2. $4x - 5y - 13 = 0$ (Equation 1)
$3x - y - 7 = 0$ (Equation 2)

Instead of solving for y in terms of x, we can also start by solving for x in terms of y.

Step 1: From Equation 1, $x=\frac{5}{4}y+\frac{13}{4}$

Step 2: Substituting the expression in Step 1 to Equation 2:

$$3\left(\frac{5}{4}y+\frac{13}{4}\right)-y-7=0$$

Step 3: Solving for y:

- $3\left(\frac{5}{4}y+\frac{13}{4}\right)-y-7=0$
- $3(5y+13)-4y-28=0$
- $15y+39-4y-28=0$
- $11y+11=0$
- $y=-1$

From Step 1:

- $x=\left(\frac{5}{4}\right)(-1)+\frac{13}{4}$
- $x=-\frac{5}{4}+\frac{13}{4}$
- $x=\frac{8}{4}=2$

The solution is **<u>(2, -1)</u>**. Check this solution by substituting back into the original system.

3. $-2x + 6y - 3 = 0$ (Equation 1)
$4x - 12y + 6 = 0$ (Equation 2)

Step 1: From Equation 1: $y=\frac{1}{3}x+\frac{1}{2}$

Step 2: $4x-12\left(\frac{1}{3}x+\frac{1}{2}\right)+6=0$

Step 3: $4x-4x-6+6=0$
$0=0$

Because 0 = 0 is a true statement, this system has **<u>infinitely many solutions</u>**. Equation 1 and Equation 2 are equivalent equations and have the same graph. This is also referred to as a **<u>dependent system</u>**.

The solution set consists of all points (x, y) lying on the line -2x + 6y – 3 = 0.

4. 3x + 9y - 8= 0 (Equation 1)
2x + 6y - 7 = 0 (Equation 2)

Step 1: From Equation 1: $x = -3y + \frac{8}{3}$

Step 2: $2\left(-3y + \frac{8}{3}\right) + 6y - 7 = 0$

Step 3: $-6y + \frac{16}{3} + 6y - 7 = 0$

$-\frac{5}{3} = 0$

Since $-\frac{5}{3} = 0$ is a false statement, we can say that the system is **<u>inconsistent</u>** and has **<u>no solution</u>**.
The graphs of the two equations will be a pair of parallel lines and because of this the system is inconsistent.

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